INVITED EDITORIAL
All LODs Are Not Created Equal

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Introduction

In recent years, the development of new methodology and/or the improvement of existing methodology in gene mapping, specifically that which is targeted toward the localization of genes underlying complex disorders, have led to a proliferation of linkage analysis techniques, most of which have been implemented in user-friendly programs and have been made freely available to the genetics community. A scan through genetic epidemiological journals reveals the almost immediate embrace of these techniques; however, user interpretation regarding the statistical properties of these tests is often insubstantial and occasionally is incorrect. To assist in this interpretation, this review provides both an overview of the underlying hypotheses and an explanation of the different statistical properties of the most-popular techniques. Furthermore, accepted thresholds for genome-wide significance and appropriate corrections for multiple testing are discussed.

Traditional LOD-Score Method

Traditional parametric linkage analysis, commonly known as “logarithm-of-odds” (LOD-score) analysis, is based on the likelihood (odds) ratio, which is the ratio (relative probability) between the probabilities of two alternatives $L_{1H}/L_{0H}$, where $L_{1H}$ is the likelihood under the null hypothesis of no linkage (recombination fraction $\theta$.5) and where $L_{1H}$ is the likelihood of the alternative hypothesis of linkage ($\theta <.5$). For a given set of pedigree genotype data, the likelihood of producing the observed data is calculated given a set of assumptions about the parameters of the underlying genetic model. These parameters include the allele frequencies of the loci involved, the relevant penetrance values, and the $\theta$ between loci. Taking the log$_{10}$ of $L_{1H}/L_{0H}$ produces the familiar LOD score (Morton 1955). The maximum LOD score (MLS) is thus obtained by testing across different values of $\theta$.

Maximum likelihood–ratio statistics are asymptotically distributed as $\chi^2$ values with degrees of freedom equal to the difference in the number of parameters estimated/maximized in the numerator compared with the number of those estimated/maximized in the denominator. In the case of the standard LOD score, $\theta$ is maximized in the numerator, whereas in the denominator $\theta$ is fixed at .5. However, when testing for linkage is done, the test is declared significant only when $0 < \theta <.5$, thereby reducing the distribution to one tail. Subsequently, the pointwise significance ($P$ value) of a LOD score can be determined using the formula $P(LOD) = .5 \times (\chi^2 > 2\ln 10 \times LOD)$, since, under the null hypothesis of no linkage, the asymptotic distribution of a LOD multiplied by twice the natural logarithm of 10 (2$\ln 10$, or 4.6) is a 50:50 mixture of a point mass at 0 and a $\chi^2$ distribution with 1 df ($\chi^2$).

Traditionally, a LOD score of $\geq 3$ has been deemed to indicate significant linkage. A LOD score of 3 indicates that the odds that the loci are linked are 1,000 times greater than the odds that they are not. However, if one randomly chooses two loci in the human genome, those loci are ~50 times (i.e., 44 autosomes plus two sex chromosomes) more likely to be on different chromosomes. Therefore, even though the data may be 1,000 times more likely to have arisen under linkage compared with nonlinkage, the fact that nonlinkages are 50 times more common than linkages implies that a LOD score of 3 corresponds to odds that are ~20:1 in favor of linkage. That is, a LOD score of 3 will prove to be spurious in ~1/20 instances ($P = .05$). Building on the earlier work of Morton (1955), Lander and Kruglyak (1995) performed calculations based on the mathematical theory of large deviations (Lander and Botstein 1989; Feingold et al. 1993), and they determined that, with the use of LOD-score analysis in human pedigrees, in which the total meiotic crossover rate between genotypes compared is set at one, an exact genomewide significance threshold of $P = .05$ is achieved at a pointwise $P$ value.

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* A Microsoft Excel spreadsheet, for performing easy calculations of $P$ values for the LOD scores described in this review, is available on request from the author.

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of $P = 4.9 \times 10^{-3}$, which is equivalent to a LOD score of $\sim 3.3$. They also introduced a pointwise $P$-value threshold of $P = 1.7 \times 10^{-3}$ (LOD score 1.86) as evidence for suggestive linkage—that is, statistical evidence expected to occur one time at random in a genome scan. Alternatively, other regions of potential interest may be identified using a nominal pointwise threshold of $P = .05$, which is equivalent to a LOD score of 0.5875.

LOD-score analysis is the most powerful linkage analysis technique when the parameters of the genetic model are known. However, the use of conventional LOD-score methods for analysis of the contribution of genetic factors to complex diseases, for which questions exist regarding disease definitions and the mode of inheritance, leads to an increase in the chance of making a false rejection of linkage (Risch 1990b). Techniques that may be used as alternatives to parametric LOD-score analyses determine allele sharing among affected relatives. Although certain assumptions about the underlying genetic model are made by these methods, allele-sharing techniques are denoted as “nonparametric” since they do not require specification of a genetic model (see Whittemore 1996). In this issue of the Journal, Ott and Hoh (2000) provide a general overview of statistical methods for disease-gene mapping.

### Allele-Sharing Methods

The most common allele-sharing technique is the affected-sib-pair (ASP) test, which compares the observed number of ASPs sharing zero, one, or two alleles identical by descent (IBD) with that which is expected under no linkage (proportioned .25:.5:.25). The general approach of ASP analysis is described in figure 1, in which, for simplicity, it is assumed that all four parental alleles can be distinguished. However, restriction of the ASP test to such “fully informative” families will undoubtedly result in a loss of information, because both parents will not always be heterozygous with different genotypes. In such cases, IBD sharing may be equivocal, with sib pairs possibly sharing or not sharing alleles, but their absolute values cannot be directly determined from the observed genotype data. Subsequently, likelihood methods were introduced to provide a satisfactory way of dealing with data for which IBD status cannot be unequivocally determined for all ASPs.

The main approach of current ASP analyses is the likelihood-ratio method of Risch (1990b, 1990c), which maximizes the likelihood of incompletely polymorphic genotype data with respect to the probabilities of sharing zero, one, or two alleles IBD. Maximization of observed sharing probabilities is performed, using an E-M (expectation-maximization) algorithm (Dempster et al. 1977), by restricting $z_1$ to .5 and $z_o \leq .25$ (or, equivalently, $z_2 \geq .25$), where $z_1$ and $z_o$ are the sharing of one and zero alleles IBD, respectively. By fixing $z_1$ at .5, the test follows an additive model and assumes no dominance variance. An additive model assumes that the genetic component of a trait’s phenotypic variance is due to the additive effects of genes at different loci. Subsequently, this model approximates genetic heterogeneity and allows for no interaction between loci (Risch 1990a). Dominance variance occurs when alleles at a locus exhibit dominance-recessive interactions, in which the effect of one allele at a locus is masked by the effect of the other allele. In the special case of siblings sharing both copies of alleles, these siblings will share any dominance relations between the two alleles. Therefore, sharing a similar dominance configuration will also increase the resemblance between siblings. Furthermore, because only one parameter ($z_o$) is maximized in the numerator, an MLS obtained in this way has the same asymptotic distribution under the null hypothesis as does the standard LOD score (Risch 1990c; Holmans 1993).

Later, Holmans (1993) and Faraway (1993) independently showed that maximization of the likelihood ratio in this way could result in parameter values that are not biologically plausible. Risch (1990b) showed that the sharing proportions ($z_o$, $z_1$, and $z_2$) can be expressed in terms of relative risks, by the equations $z_o = .5\lambda_o/\lambda_s$ and $z_2 = .25\lambda_o/\lambda_s$. The $\lambda_o$, $\lambda_s$, and $\lambda_m$ symbols represent the relative risk for a sibling, offspring, and MZ twin of a proband, respectively, where $\lambda_{sib}$ denotes the risk (affection probability) to the sibling/
offspring/MZ twin of a proband, divided by the population prevalence. With relative risks expressed in terms of population and dominance variances, it can be shown that \( \lambda_s \leq \lambda \), and \( \lambda_m \geq \lambda \) (James 1971; Risch 1990a), thus restricting the set of valid sharing proportions to a triangle in the \((z_o, z_1)\) plane, bounded by the lines \( z_o = 0 \), \( z_1 = 0.5 \), and \( z_1 = 2z_o \). Therefore, Holmans and Faraway suggested that maximization be restricted to the set of sharing probabilities consistent with these possible genetic models; denoted as the “possible triangle” (PT), the set is defined by \( z_1 \leq 0.5 \) and \( z_o \leq 0.5 \times z_1 \).

Restriction to the PT follows a multiplicative model—representing epistasis (interaction) among loci—allows for dominance variance, and has been shown to increase the power of the likelihood-ratio test (Holmans 1993; Holmans and Clayton 1995). Hence, this approach would seem to be more suited to the analysis of complex genetic disorders, for which multiple interacting genes are thought to underlie susceptibility. MLSs obtained in this way (\( MLS_{PT} \)) are mixtures of \( \chi^2 \) distributions with 0, 1, and 2 df under the null hypothesis. The exact mixing proportions are complicated and depend on allele frequencies, but they are approximately \( 0.40z_o^2 \), \( 0.5z_1^2 \), and \( 0.08z_1 \) (Holmans 1993; Knapp 1997, pp. 150–151; Sham 1998, p. 110). Hence, to obtain the nominal threshold of \( P < 0.05 \), an \( MLS_{PT} > 0.74 \) is required, where \( P(MLS_{PT}) = 0.5 \times P(\chi_1^2 > 2 \ln 10 \times MLS_{PT}) + 0.08 \times P(\chi_2^2 > 2 \ln 10 \times MLS_{PT}) \). Moreover, when allele-sharing methods are used in human sib pairs, for which the crossover rate is 2 (i.e., LOD-score analysis in pedigrees concentrates on inheritance between two parents and one child, whereas ASP analysis has twice the potential for crossover, because inheritance from the parents now involves two children), evidence for genomewide suggestive (\( P = 7.4 \times 10^{-4} \)) and significant (\( P = 2.2 \times 10^{-10} \)) linkage (Lander and Kruglyak 1995) is reached at \( MLS_{PT} \) values of 2.45 and 3.93, respectively. The popular ASP analysis programs ASPEX (Hauser et al. 1996), MAPMAKER/SIBS (Kruglyak and Lander 1995), and SPLINK (Holmans 1993; Holmans and Clayton 1995) perform these ASP tests.

Although, because of their obvious inheritance pattern, X-linked disorders are normally easier to analyze than are autosomal disorders, evidence for heterogeneity and uncertainties regarding penetrance values may make a “nonparametric” approach desirable. Additionally, when a genome scan is performed, it would be useful to ascertain the significance of X-linked markers, in a manner analogous to that used for autosomal markers. Cordell et al. (1995) extended Holmans’ method to X-linked data, showing that it was not appropriate to combine data from brother-brother (bb), sister-sister (ss), and brother-sister (bs) pairs; instead, they introduced a statistic that considers the three groups separately by restricting the maximization to the following genetically valid values: \( 0 \leq z_{bb} \leq 0.5 \), \( 0 \leq z_{ss} \leq 0.5 \), and \( 0 \leq z_{bs} \leq 0.5 \), where \( z_i \) represents IBD sharing of maternal alleles. The sum of these three MLS statistics, known as “\( X-MLS_{PT} \),” is a mixture of \( \chi^2 \) distributions with 1, 2, and 3 df, where

\[
\begin{align*}
& P(X - MLS_{PT}) = 0.375 \times P(\chi_1^2 > 2 \ln 10 \times X - MLS_{PT}) \\
& \quad + 0.375 \times P(\chi_2^2 > 2 \ln 10 \times X - MLS_{PT}) \\
& \quad + 0.125 \times P(\chi_3^2 > 2 \ln 10 \times X - MLS_{PT}).
\end{align*}
\]

Therefore, to obtain the nominal threshold of \( P < 0.05 \), an \( MLS_{PT} > 1.18 \) is required, whereas evidence for suggestive and significant linkage is reached at \( MLS_{PT} \) values of 3.06 and 4.62, respectively. Only MAPMAKER/SIBS (Kruglyak and Lander 1995) has the PT restriction implemented in this manner. Other ASP programs analyze X-linked data by simply summing all maternal IBD sharing together, so that, under the null hypothesis, \( z_o = z_1 = 0.5 \), and the likelihood is maximized with respect to only one parameter (i.e., \( z_o < 0.5 \) or, equivalently, \( z_1 > 0.5 \)), thus giving this (X-)MLS the same properties as the standard LOD score.

Another powerful ASP technique is the mean \((t_i)\) test (Blackwelder and Elston 1985), which measures the mean proportion of IBD allele sharing and compares it with that which is expected under the null hypothesis (50%). The test statistic has, asymptotically, a standard normal distribution under the null hypothesis, and, because the alternative hypothesis of linkage is given when IBD sharing is only 50%, the test is one sided. Therefore, accurate \( P \) values can be obtained by use of a one-sided \( t \)-test, as implemented in the SIBPAL program (S.A.G.E.). It was later shown by Knapp et al. (1994) that the mean test is equivalent to a parametric LOD score obtained under a recessive model, in which the parents are either phenotype “unknown” or (equivalently) unaffected and in which the penetrance is set close to zero. The SIBPAIR program is based on this technique and thus produces a standard LOD score.

Other popular allele-sharing analysis programs include GENEHUNTER (Kruglyak et al. 1996), GENEHUNTER-PLUS (Kong and Cox 1997), and ALLEGRO (Gudbjartsson et al. 2000), which are also capable of analyzing allele sharing between more-distant relatives by counting all possible inheritance vectors. By calculation of the inheritance-vector distribution conditioned on observed genotype data, the probability distribution is concentrated on certain inheritance vectors—that is, the (likely) inheritance (IBD) pattern(s) is (are) determined (Lander and Green 1987). Figure 2 demonstrates how analysis of allele sharing can be extended to multiple relatives.
Figure 2  
A, Nuclear family with two affected siblings—identical to that shown in figure 1. 
B, Nuclear family with three affected siblings. 
Both pedigrees show the possible genotype configurations for a marker where all four parental alleles can be distinguished. The genotypes are denoted as “1,” “2,” “3,” and “4,” and are written in paternal/maternal order. These genotype configurations may also be represented as vectors, where 0 or 1 represents transmission of the paternal or maternal allele, respectively. Representation of inheritance patterns as vectors facilitates their interpretation by computer programs. For example, in panel A, (0,0,0,0) indicates that both siblings have inherited the (grand)paternal allele from both parents, whereas (1,1,1,1) indicates that both siblings have inherited the (grand)maternal allele from both parents. Also, (0,0,0,1) and (0,1,0,0) both indicate that one sibling has inherited one (grand)paternal allele from the father and one (grand)maternal allele from the mother, whereas the other sibling has inherited the (grand)paternal alleles from both parents. Inheritance-vector probabilities may be calculated by enumerating over all possibilities and may subsequently be used in tests examining allele sharing between any set of affected relatives.
GENEHUNTER calculates a nonparametric linkage (NPL) score on the basis of either the $S_{\text{pairs}}$ (Weeks and Lange 1988) or $S_{\text{all}}$ statistic (Whittemore and Halpern 1994). $S_{\text{pairs}}$ measures the number of alleles shared IBD by a pair of affected relatives, whereas $S_{\text{all}}$ captures the observed IBD probability over all possible configurations and puts extra weight on families in which three or more affected individuals share the same allele. For sibships with only two affected siblings, the two statistics are equal, but, for $S_{\text{all}}$, sharply increasing weight is given to families as the number of affected individuals sharing a particular allele increases. The $S_{\text{all}}$ statistic is best suited to analysis of dominant loci, since more affected individuals per family are likely to share the same allele with this type of inheritance. Under the null hypothesis and when many similar pedigrees are analyzed, an NPL score is a normally distributed statistic with mean zero and variance one; however, when calculated from a small sample size and/or from pedigrees differing in size, the asymptotic properties of the NPL statistic are unreliable. Therefore, sample-specific NPL significance (an exact $P$ value) is determined using simulation, by means of enumerating (counting of) all possible inheritance vectors and by obtaining the NPL-score distribution under no linkage. However, when inheritance information is imperfect, the variance of the NPL score is <1, thus making the test conservative. Put simply, because the true inheritance pattern is not always known with certainty but significance is evaluated using the null distribution expected in the case of complete informativeness, the NPL score will tend to be conservative, since it will be averaged across the likely inheritance patterns. The degree of conservativeness depends on the number of untyped individuals, marker heterozygosity, and, when multipoint calculations are performed, marker spacing.

To resolve the conservativeness of the NPL statistic when IBD information is incomplete, Kong and Cox (1997) proposed a likelihood approach, which has subsequently been implemented in the GENEHUNTER-PLUS (Kong and Cox 1997) and ALLEGRO (Gudbjartsson et al. 2000) programs. These programs both produce nonparametric LOD scores by maximization of a single parameter ($\delta$) in the numerator, on the basis of observed genotype data. The $\delta$ represents the degree of allele sharing in a pedigree, where, under the null hypothesis, $\delta = 0$ and $\delta > 0$ corresponds to the alternative of excess sharing. Therefore, these nonparametric LOD scores are asymptotically distributed in the same manner as the standard LOD score.

### Table 1

<table>
<thead>
<tr>
<th>Significance Threshold</th>
<th>LOD/MLS</th>
<th>MLS$_{\text{het}}$</th>
<th>X-MLS$_{\text{het}}$</th>
<th>MMLS/het</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P &lt; .05$</td>
<td>.59</td>
<td>.74</td>
<td>1.18</td>
<td>1.09</td>
</tr>
<tr>
<td>$P &lt; .01$</td>
<td>1.18</td>
<td>1.38</td>
<td>1.90</td>
<td>1.71</td>
</tr>
<tr>
<td>$P &lt; .005$</td>
<td>1.44</td>
<td>1.66</td>
<td>2.21</td>
<td>1.99</td>
</tr>
<tr>
<td>$P &lt; .001$</td>
<td>2.07</td>
<td>2.32</td>
<td>2.93</td>
<td>2.63</td>
</tr>
<tr>
<td>$P &lt; 7.4 \times 10^{-4}$</td>
<td>2.19</td>
<td>2.45</td>
<td>3.06</td>
<td>2.75</td>
</tr>
<tr>
<td>$P &lt; 2.2 \times 10^{-3}$</td>
<td>3.63</td>
<td>3.93</td>
<td>4.62</td>
<td>4.20</td>
</tr>
<tr>
<td>$P &lt; 3 \times 10^{-2}$</td>
<td>5.3</td>
<td>5.76</td>
<td>6.52</td>
<td>5.99</td>
</tr>
</tbody>
</table>

* $a$ Suggestive linkage threshold when allele-sharing methods in human sib pairs are used (Lander and Kruglyak 1995).

* $b$ Significant linkage threshold when allele-sharing methods in human sib pairs are used (Lander and Kruglyak 1995).

* $c$ Highly significant linkage threshold when allele-sharing methods in human sib pairs are used (Lander and Kruglyak 1995).

### Extended LOD-Score Methods

Recently, there have been a number of studies indicating that LOD-score (parametric) models can be more powerful than “nonparametric” techniques, as long as the mode of inheritance is at least approximately correct. In particular, LOD scores are calculated under a simple dominant (e.g., 50% penetrant) as well as a simple recessive model (Vieland et al. 1992; Durner et al. 1999), with allowance for locus heterogeneity (Barrett et al. 1999), after which the highest score is reported (Greenberg 1989; Hodge et al. 1997). However, heterogeneity (admixture) LOD (HLOD) scores obtained in this way, whether single- or multipoint, are a mixture of $\chi^2$ distributions with 1 and 2 df, resulting from the introduction of an additional free (heterogeneity) parameter. In addition, these scores require a correction for the testing of two inheritance models. The $\chi^2$ distribution mixtures are quite complicated; instead, a Bonferroni-type correction (Terwilliger and Ott 1994, pp. 237–238), which is approximately accomplished by adding .3 (i.e., $\log_{10} 2$) to the significant linkage threshold, is suggested. Alternatively, the standard LOD-score pointwise asymptotic $P$ value should be doubled. Analogously, the $P$ value should again be doubled to correct for the testing of two inheritance models, since testing the same data with two inheritance models increases the type I error. Clearly, the use of $P$ values greatly facilitates adjustment for such multiple-hypothesis testing. Subsequently, analyses using this “MMLS/het” statistic require a corrected pointwise nominal $P$ value of $< 0.0125$ (MMLS/het $> 1.09$). Furthermore, $P < 1.85 \times 10^{-4}$ (MMLS/het $> 2.75$) and $P < 5.5 \times 10^{-6}$ (MMLS/het $> 4.2$) are required for suggestive and significant linkage, respectively.
Conclusions

Although only a few different statistical tests have been outlined in this review, it has been clearly demonstrated that LOD scores may vary greatly in their statistical properties and, therefore, in their level of significance. To graphically demonstrate this, the required LOD-score values for common significance thresholds have been calculated (table 1) for the different statistical techniques discussed here. As shown in table 1, the differences can be quite substantial, with some thresholds differing by 1 LOD unit. Moreover, because of these differences, care must be taken to quote the relevant genomewide significance thresholds for the analysis technique used.

Geneticists have traditionally appreciated the use of LOD scores more than that of P values; this is no more obvious than when P values are converted to LOD scores. However, given the plethora of linkage statistics currently available to the genetics community, it is appropriate to reassess the general format that one uses to report linkage results. In conclusion, it is strongly recommended that, to aid in the interpretation of linkage results, the appropriate significance thresholds plus both the LOD score and, most important, the corresponding P value should be reported in all linkage studies.

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Electronic-Database Information

The URLs for data in this article are as follows:

Alphabetical List of Genetic Analysis Software, http://linkage.rockefeller.edu/soft/list.html (for all programs referenced in this review [plus many others])

ASPEX package, ftp://lahmed.stanford.edu/pub/asplex

S.A.G.E. (Statistical Analysis for Genetic Epidemiology), release 2.2, http://darwin.mhmc.cwru.edu/pub/sage.html (for S.A.G.E. computer package available from the Department of Epidemiology and Biostatistics, Case Western Reserve University, Cleveland)

SIBPAIR, http://linkage.cpmc.columbia.edu/software/analyze (program for sib-pair analysis on nuclear families)

References


